A Bogus Dichotomy

In math, basic arithmetic skills and conceptual understanding are completely intertwined.

By Hung-Hsi Wu

Education seems to be plagued by false dichotomies.

Until recently, when research and common sense gained the upper hand, the debate over how to teach beginning reading was characterized by many as "phonics vs. meaning." It turns out that, rather than a dichotomy, there is an inseparable connection between decoding — what one might call the skills part of reading — and comprehension.

Fluent decoding, which for most children is best ensured by the direct and systematic teaching of phonics and lots of practice reading, is an indispensable condition of comprehension.

"Facts vs. higher-order thinking" is another example of a false choice that we often encounter these days, as if thinking of any sort — high or low — could exist outside of content knowledge.

In mathematics education, this debate takes the form of "basic skills or conceptual understanding." This bogus dichotomy would seem to arise from a common misconception of mathematics held by a segment of the public and the education community: that the demand for precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding.

The truth is that in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding. There is not "conceptual understanding" and "problem-solving skill" on the one hand and "basic skills" on the other. Nor can one acquire the former without the latter.

It has been said that had Einstein been born at the time of the Stone Age, his genius might have enabled him to invent basic arithmetic, but probably not much else.

However, because he was born at the end of the 19th century — with all the techniques of advanced physics at his disposal — he created the theory of relativity. And so it is with mathematics. Conceptual advances are invariably built on the bedrock of technique.

Without the quadratic formula, for example, the theoretical development of polynomial equations and hence of algebra as a whole would have been very different.

The ability to sum a geometric series, something routinely taught in Algebra II, is ultimately responsible for the theory of power series, which lurks inside every calculator.

The analogue of the same phenomenon in the artistic domain is even more transparent. A violinist who still worries about fingering positions cannot hope to impress with beauty of tone or elegance of phrasing, and an opera singer without the requisite high notes would try in vain to stir our souls with searing passion.

In good art, as in good mathematics, technique and conception go hand in hand.

The desire to achieve understanding in a technical subject such as mathematics while minimizing the component of skills is a most human one. Such a desire cannot be indulged, however, without doing great harm to students' education. There are many reasons.

Sometimes a simple skill is absolutely indispensable for the understanding of more sophisticated processes. For example, the familiar long division of one number by another provides the key ingredient to understanding why fractions are repeating decimals.

Then there's the fact that the arithmetic of ordinary fractions (adding, multiplying, reducing to lowest terms, etc.) develops the necessary pattern for understanding rational algebraic expressions.

At other times, it is the *fluency* in executing a basic skill that is essential for further progress in the course of one's mathematics education. The automaticity in putting a skill to use frees up mental energy to focus on the more rigorous demands of a complicated problem.

Finally, when a skill is bypassed in favour of a conceptual approach, the resulting conceptual understanding often is too superficial. This happens with almost all current attempts at facilitating the teaching of fractions.

It remains to make a passing comment on the idea of skipping the standard algorithms by asking children to invent their own algorithms instead. The justification is that inventing algorithms promotes conceptual understanding.

What is left unsaid is that when a child makes up an algorithm, the act raises two immediate concerns: one is whether the algorithm is correct and the other is whether it is applicable under all circumstances. In short, correctness and generality.

In a class of, say, 30 students, for the teacher to carefully check 30 new algorithms every time is a Herculean task. More likely than not, some incorrect or non-generalizable algorithms would slip through, and some children would come out of this encounter with mathematics with no understanding at all.

As Euclid told King Ptolemy in the fourth century, BC, there is no royal road to geometry. Neither is there a royal road to conceptual understanding. Let us teach our children mathematics the honest way by teaching both skills and understanding.

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