

Manipulative Education

Must all mathematical concepts be taught with manipulatives?

By John Mighton

For some time now, educators have advocated that we move away from the rote learning of rules and operations. This is a very positive development in education. Students should understand why rules work and how they are connected to the world.

But unfortunately, in arguing against rote learning, some educators have set up a false dichotomy between mathematical rules and operations on the one hand and concepts and models on the other.

Not all concepts in mathematics are concrete (as the case of negative multiplication illustrates). And if a rule is taught without reference to a model, it is not necessarily taught in a rote way.

Whenever a child sees a pattern in a rule, or applies a rule to a case he has never encountered, he is doing math conceptually, even if he hasn't consulted a model in his work (and even if he hasn't discovered the rule himself).

I read in an educational journal recently that when a child uses a rule to find an answer to a problem, the child isn't thinking. I was surprised to learn this, as most of the work I did as a graduate math student consisted of following rules. Every time I used a rule to solve a problem I hadn't seen before, I had the distinct impression that I was thinking.

A common dogma of contemporary education is that a teacher who neglects to use concrete materials (such as pattern blocks or fraction strips) whenever he introduces a mathematical idea is not teaching "conceptually".

Concrete materials are very useful tools for teaching mathematics. There are topics in elementary math, such as the classification of three-dimensional solids, which are hard to teach without physical models.

But many topics in elementary math can also be taught more abstractly.

Young children have no fear of

the symbolic side of mathematics: they are much more open-minded, and more fascinated by patterns and puzzles, than most adults. If children were taught to excel at the symbolic game of math at an earlier age, they might not encounter the problems that most students face in high school.

It can take a great deal of time (relative to the amount of learning that takes place) to conduct a lesson with manipulatives. While it is important that students receive some lessons with manipulatives, students often learn as much math from drawing a simple picture as they do from playing with a manipulative.

In mathematics, the ability to draw a picture or create a model in which only the essential features of a problem are represented is an essential skill.

Concrete materials do not, as is widely believed, display their interpretations on their surface. You can't simply hand out a set of manipulatives to a group of children and expect the majority to use them to derive efficient rules and operations. Children usually need a great deal of guidance in order to deduce anything significant from playing with concrete materials.

The line between abstract and concrete thought is often rather fuzzy: even the simplest manipulatives and models do not provide transparent representations of mathematical concepts.

I once saw kids in a remedial class reduced to tears when their teacher tried to introduce the operation of addition using base-ten materials. When I showed the children how to add (and how to subtract, multiply, and divide) by counting up on their fingers, they were able to perform the operations instantly.

In my experience, the hand is the most effective (and cheapest) manipulative for students who have serious learning difficulties.

When children perform operations by counting or skip counting on

their fingers, they get a sense of the positions of the numbers in their body. I've yet to meet a child, even as early as grade 1, who couldn't do all of the operations required for our grade 7 fractions unit on their fingers.

(Of course, children should eventually be weaned off using their fingers. And base-ten materials are very useful for teaching more advanced concepts, such as place value. But in the early phases of a math program, I recommend teaching weaker students who need to catch up to perform basic operations on their fingers.)

I believe students would cover far more material in a year if we could find a better balance between symbolic and concrete work in our curriculum. Finding this balance may prove difficult, however, as schools are being pushed by educational experts to include more manipulatives in their math programs.

I recently showed an influential educator the results of a pilot study that I was very proud of: after a month of instruction, an entire grade 3 class that I taught (including several slow learners) had scored over 90% on a grade 7 test on operations with fractions.

On seeing the tests, the educator said they made her blood boil. I expect she was so upset because I wasn't supposed to be teaching fractions without models in grade 3. This episode (and many other recent encounters) showed me the extent to which educators have come to associate mathematical concepts with concrete materials.

(Adapted with permission from JUMP at Home, a series of excellent math workbooks available at good bookstores everywhere. For a more detailed discussion on this issue, see the JUMP at Home workbooks. Dr. Mighton is a mathematician and award-winning playwright.)